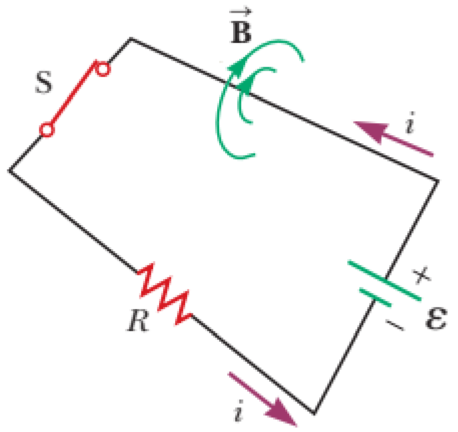
Unidad 5: Inductancia

Circuitos de corriente alterna

Inductancia



Ley de Faraday

$$\varepsilon = -\frac{d\phi_B}{dt}$$

f.e.m autoinducida

Autoinducción

$$\varepsilon_L = -L\frac{di}{dt} \qquad L: inductancia$$

$$\varepsilon_L = -N\frac{d\phi_B}{dt} \Rightarrow L = \frac{N\phi_B}{i} \qquad L = \frac{\varepsilon_L}{\frac{di}{dt}} \qquad R = \frac{\Delta V}{I}$$

La inductancia presenta un efecto similar al de la resistencia

$$Unidades\ para\ L \rightarrow [Hy]; \left[\frac{V.s}{A}\right]$$

Para el caso de una bobina

$$\phi_B = B.A = \mu_0 nIA = \mu_0\frac{N}{l}IA \Rightarrow L = \frac{N\phi_B}{i} = \frac{\mu_0 N^2 A}{l}$$

Caso

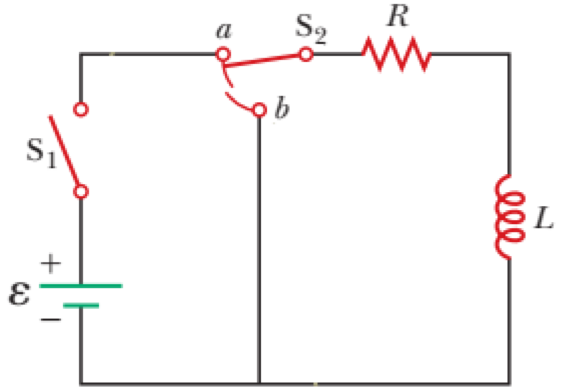
Si consideramos una bobina de 280 vueltas, 290 mm de longitud y una sección de 5 cm2. Calcular la inductancia y la f.e.m autoinducida si la corriente disminuye a 40 A/s.

$$L = \frac{\mu_0 N^2 A}{l} = 4\pi \times 10^{-7} \left[\frac{T.m}{A}\right] \frac{280^2}{0{,}290[m]} 5 \times 10^{-4} [m^2] = 0{,}1698[mH]$$

$$\varepsilon_L = -L \frac{di}{dt} \quad = -0{,}0001698[H].(-40) \left[\frac{A}{s}\right] = 6{,}792[mV]$$

Autoinductancia

Circuitos RL



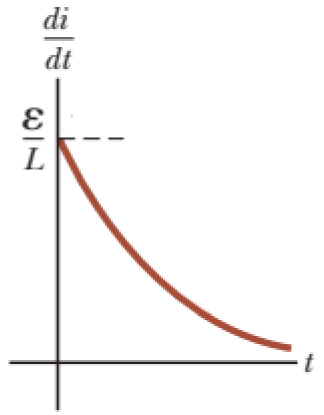
$$\varepsilon - iR - L\frac{di}{dt} = 0$$

$$\frac{di}{dt} + i\frac{R}{L} = \frac{\varepsilon}{L} \Rightarrow \frac{di}{dt} = \frac{\varepsilon}{L} - i\frac{R}{L}$$

$$\frac{di}{dt} = \frac{R}{L}\left(\frac{\varepsilon}{R} - i\right) \Rightarrow \frac{di}{\left(\frac{\varepsilon}{R} - i\right)} = \frac{R}{L}dt \Rightarrow d\left(\frac{\varepsilon}{R} - i\right) = -di$$

$$\frac{d\left(\frac{\varepsilon}{R} - i\right)}{\left(\frac{\varepsilon}{R} - i\right)} = -\frac{R}{L}dt \Rightarrow \ln\left(\frac{\varepsilon}{R} - i\right) = -\frac{R}{L}t + \ln K \Rightarrow \frac{\varepsilon}{R} - i = Ke^{-\frac{R}{L}t} \Rightarrow i = \frac{\varepsilon}{R} - Ke^{-\frac{R}{L}t}$$

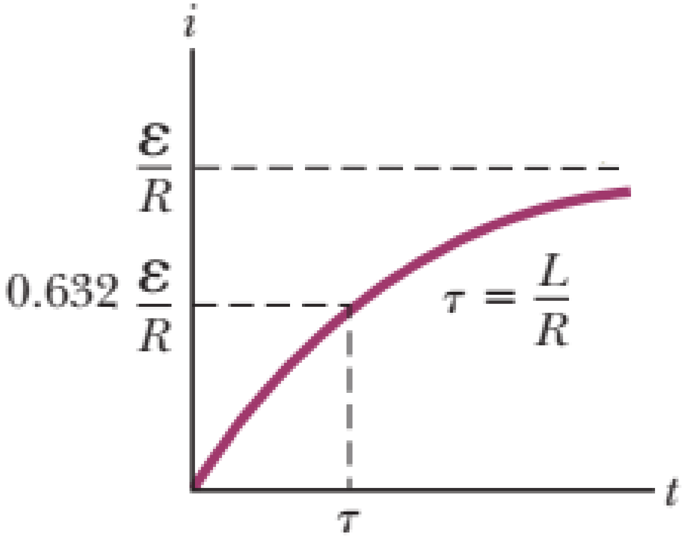
$$Para\ t = 0 \rightarrow i = 0 \qquad K = \frac{\varepsilon}{R} \Rightarrow \qquad i = \frac{\varepsilon}{R}\left(1 - e^{-\frac{R}{L}t}\right)$$

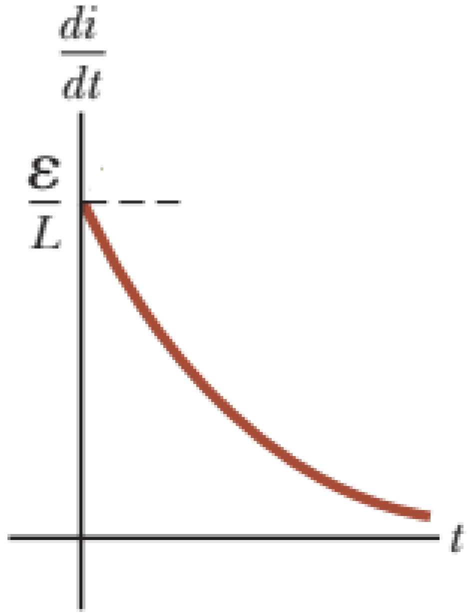


Circuitos RL

Constante de Tiempo

$$\tau = \frac{L}{R}[s]$$





Caso circuito RL

Consideremos un circuito RL donde la fuente tiene $\varepsilon = 12[V], R = 5[\Omega], L = 40[mH]$

$$i = \frac{\varepsilon}{R}\left(1 - e^{-\frac{R}{L}t}\right) = \frac{12[V]}{5[\Omega]}\left(1 - e^{-\frac{5[\Omega]}{40\times 10^{-3}[H]}t}\right) = 2{,}4(1 - e^{-125t})[A]$$

$$t = 2[ms] \qquad i = 2{,}4(1 - e^{-125t})[A] = 0{,}53[A]$$

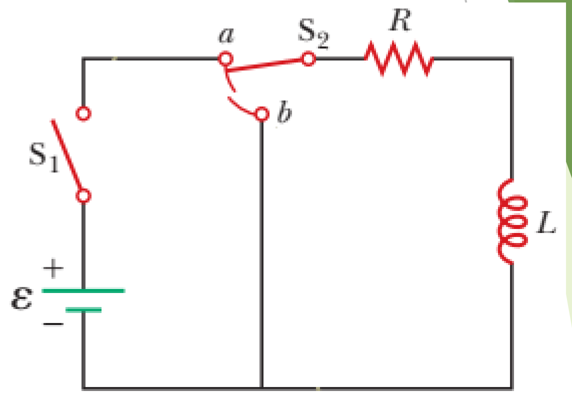
$$\tau = \frac{L}{R} = \frac{40 \times 10^{-3}[H]}{5[\Omega]} = 0{,}008[s]$$

Energía de un campo magnético

$$\varepsilon = iR + L\frac{di}{dt}$$

$$i\varepsilon = i^2R + Li\frac{di}{dt}$$

$$\frac{dU_B}{dt} = Li\frac{di}{dt}$$



$$U_B = \int dU_B = \int_0^i Li\,di = L\int_0^i i\,di$$

$$U_B = \frac{1}{2}Li^2$$

$$L = \mu_0 n^2 V$$

$$B = \mu_0 n i$$

$$i = \frac{B}{\mu_0 n}$$

V: volumen de la bobina

$$U_B = \frac{1}{2}\mu_0 n^2 V\left(\frac{B}{\mu_0 n}\right)^2 = \frac{B^2}{2\mu_0}V$$

$$u_B = \frac{U_B}{V}$$

$$u_B = \frac{B^2}{2\mu_0}$$

Inductancia mutua

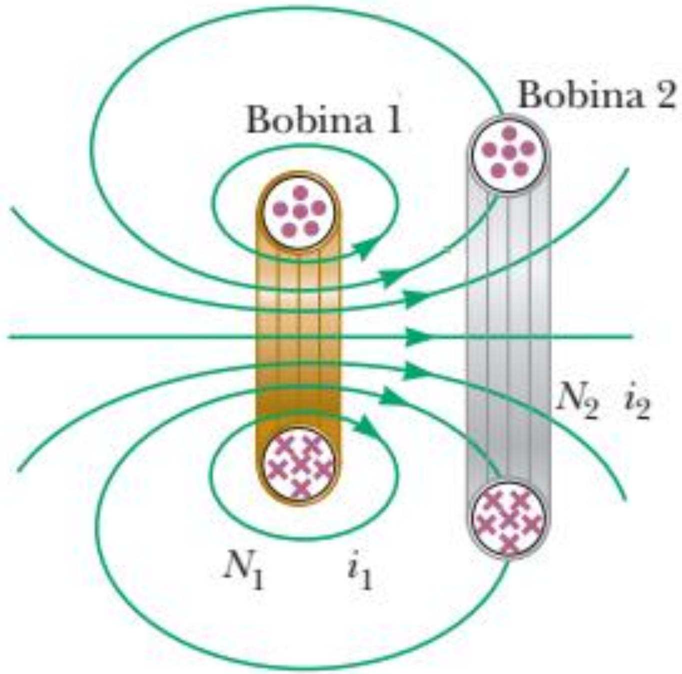
$$\varepsilon = -N\frac{d\phi_B}{dt} \qquad L = \frac{N\phi_B}{i}$$

$$M_{12} = \frac{N_2\phi_{12}}{i_1}$$

$$\varepsilon_2 = -N_2\frac{d\phi_{12}}{dt} \quad = -N_2\frac{d}{dt}\left(\frac{M_{12}i_2}{N_2}\right) = -M_{12}\frac{di_1}{dt}$$

$$\varepsilon_1 = -M_{21}\frac{di_2}{dt} \qquad M_{12} = M_{21}$$

$$\varepsilon_1 = -M\frac{di_2}{dt} \qquad \varepsilon_2 = -M\frac{di_1}{dt}$$

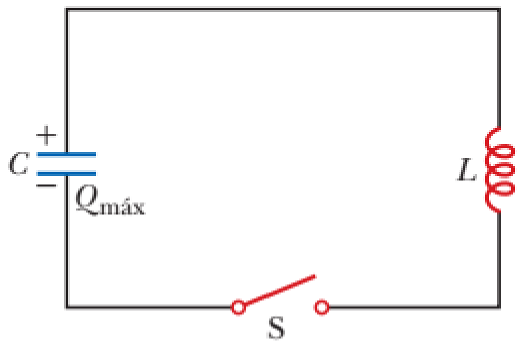


Oscilaciones en un circuito LC

Previo al cierre del interruptor, el capacitor se encuentra cargado.

$$t < 0 \quad U_C = \frac{Q_{max}^2}{2C} \quad U_B = 0$$

$$t = 0 \quad U_C = \frac{Q^2}{2C} \to 0 \quad U_B = \frac{1}{2}Li^2$$



Oscilaciones en un circuito LC

Energía almacenada en un circuito LC

$$U = U_C + U_B = \frac{q^2}{2C} + \frac{1}{2}Li^2 = \frac{Q_{max}^2}{2C} \Rightarrow \frac{dU}{dt} = \frac{d}{dt}\left(\frac{q^2}{2C} + \frac{1}{2}Li^2\right) = \frac{q}{C}\frac{dq}{dt} + Li\frac{di}{dt} = 0$$

$$i = \frac{dq}{dt} \Rightarrow \frac{di}{dt} = \frac{d^2q}{dt^2} \Rightarrow \frac{q}{C}\frac{dq}{dt} + Li\frac{d^2q}{dt^2} = 0 \Rightarrow \frac{d^2q}{dt^2} = -\frac{1}{LC}q$$

$$q = Q_{max}\cos(\omega t + \varphi) \qquad \omega = \frac{1}{\sqrt{LC}}$$

$$i = \frac{dq}{dt} = -\omega Q_{max}\sin(\omega t + \varphi)$$

Para determinar el valor φ consideramos las condiciones iniciales, donde para $t = 0 \rightarrow i = 0$

$Por\ lo\ tanto:\ \varphi = 0.$

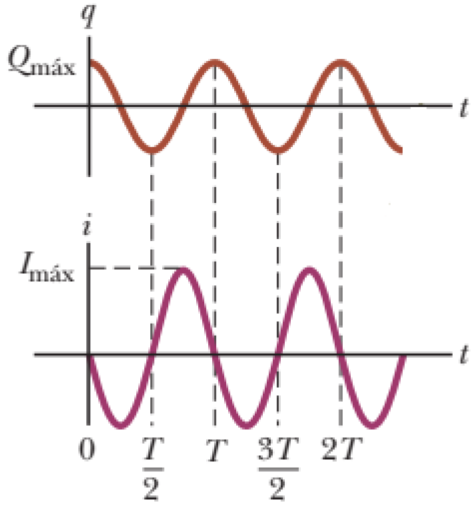
Oscilaciones en un circuito LC

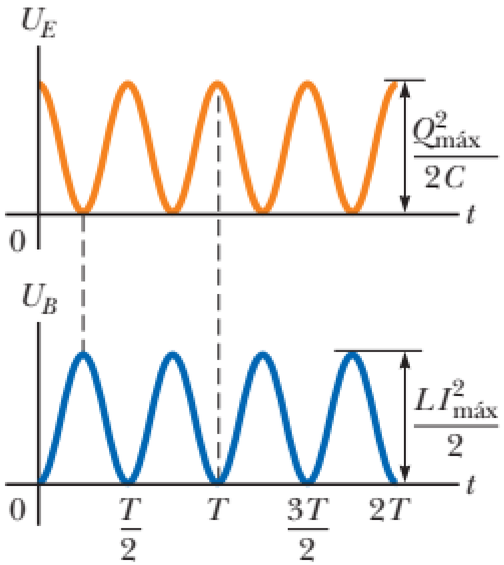
$q = Q_{max} \cos \omega t$ $\qquad$ $i = -\omega Q_{max} \sin \omega t = -I_{max} \sin \omega t$

$$U = U_C + U_B = \frac{Q_{max}^2}{2C} cos^2\ \omega t + \frac{1}{2} L I_{max}^2\ sin^2\ \omega t$$

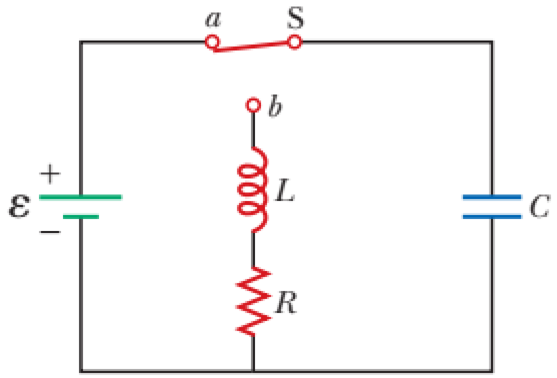
$$cos^2\ \omega t + sin^2\ \omega t = 1$$

$$\frac{Q_{max}^2}{2C} = \frac{1}{2} L I_{max}^2$$





Circuito RLC



$$\frac{dU}{dt} = -i^2R \qquad U = U_C + U_B$$

$$\frac{q}{C}\frac{dq}{dt} + Li\frac{di}{dt} + i^2R = 0 \qquad i = \frac{dq}{dt}$$

$$Li\frac{d^2q}{dt^2} + i^2R + \frac{q}{C}i = 0 \qquad \frac{d^2q}{dt^2} + \frac{R}{L}\frac{dq}{dt} + \frac{q}{LC} = 0$$

→ Ecuación Diferencial, 2°Orden, homogénea

$$D^2 + \frac{R}{L}D + \frac{1}{LC} = 0$$

→ Ecuación característica

Circuito RLC

$$D^2 + \frac{R}{L}D + \frac{1}{LC} = 0$$

$$D_1 = \frac{-\frac{R}{L} + \sqrt{\left(\frac{R}{L}\right)^2 - \frac{4}{LC}}}{2}$$

$$D_2 = \frac{-\frac{R}{L} - \sqrt{\left(\frac{R}{L}\right)^2 - \frac{4}{LC}}}{2}$$

$$D_1 = \alpha + \beta$$

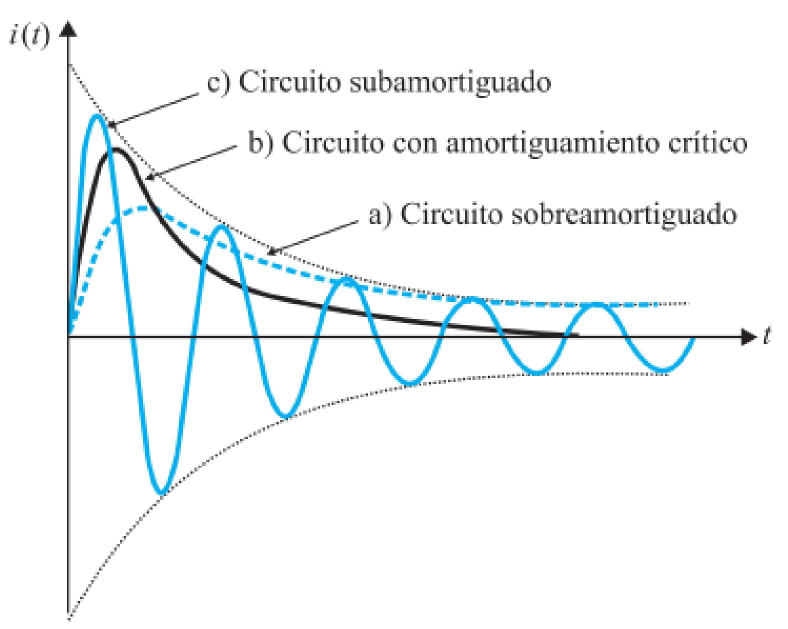
$$D_2 = \alpha - \beta$$

Circuito RLC

$Si\left(\frac{R}{L}\right)^2 > \frac{4}{LC}, las\ raíces\ de\ D_1\ y\ D_2\ son\ reales\ y\ distintas \rightarrow$
$Oscilación\ sobreamortiguada.$

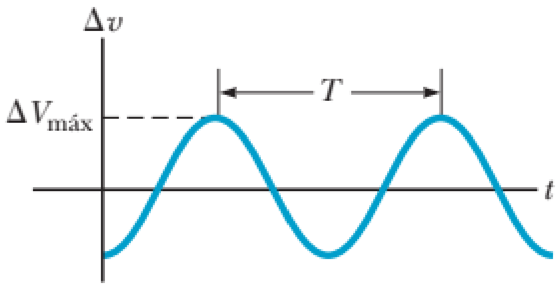
$Si\left(\frac{R}{L}\right)^2 = \frac{4}{LC}, las\ raíces\ de\ D_1\ y\ D_2\ son\ reales\ y\ distintas$
$\rightarrow Oscilación\ crítica.$

$Si\left(\frac{R}{L}\right)^2$
$< \frac{4}{LC}, las\ raíces\ de\ D_1\ y\ D_2\ no\ son\ reales\ y\ distintas$
$\rightarrow Oscilación\ subamortiguada$



Fuentes de corriente alterna

$$\Delta v = \Delta V_{max} \sin(\omega t + \varphi)$$



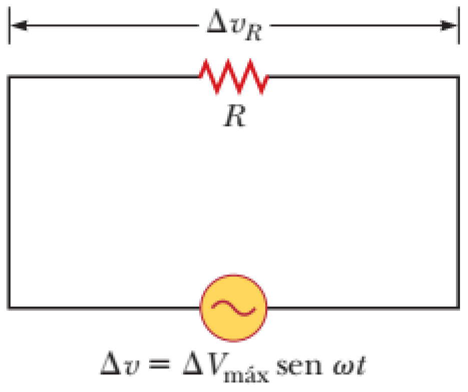
ΔV_{max}: amplitud máxima de la onda senoidal

$$\omega = 2\pi f = \frac{2\pi}{T} \qquad f = \frac{1}{T}$$

Resistores en un circuito de CA

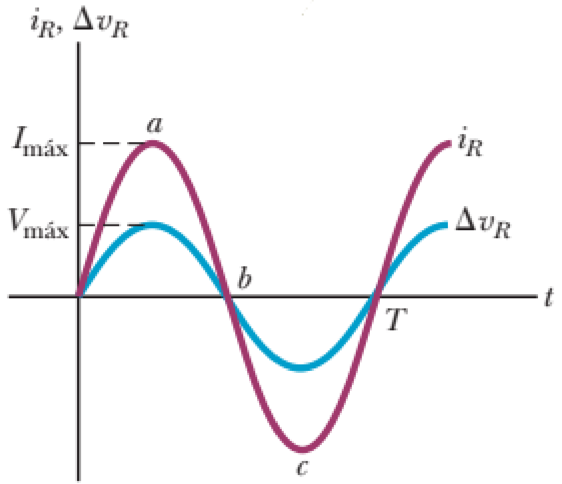
$$\Delta v - iR = 0$$

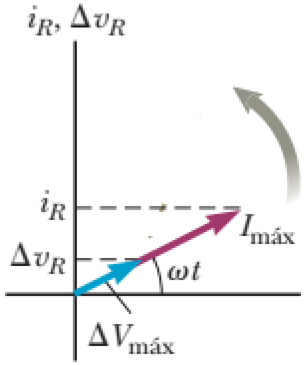
$$i = \frac{\Delta v}{R} = \frac{\Delta V_{máx}}{R}\sin(\omega t + \varphi) \qquad \varphi = 0°$$



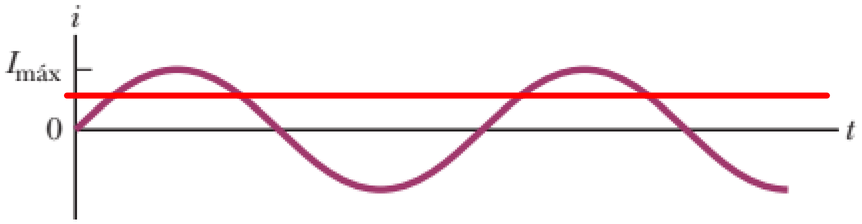
$$I_{max} = \frac{\Delta V_{máx}}{R}$$

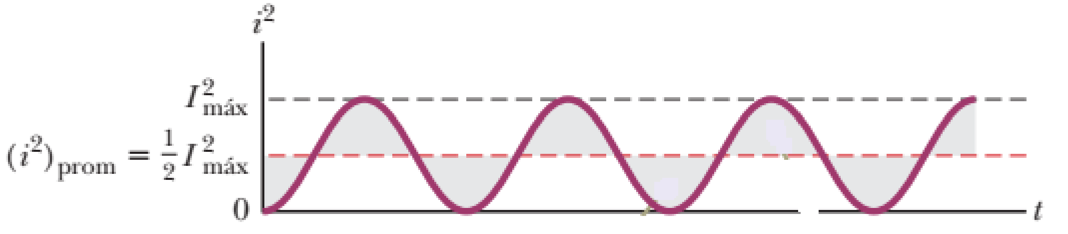
$$\Delta v_R = i_R R = I_{máx} R \sin \omega t$$





Resistores en un circuito de CA





$$P = i^2 . R$$

$$I_{ef}^2 = \frac{1}{T}\int i^2 dt \Rightarrow I_{ef} = \sqrt{\frac{1}{T}\int i^2 dt} \Rightarrow I_{ef} = \frac{I_{max}}{\sqrt{2}} = 0{,}707 I_{max}$$

Inductores en un circuito de CA

$$\Delta v - \Delta v_L = 0 \qquad \Delta v - L\frac{di_L}{dt} = 0$$

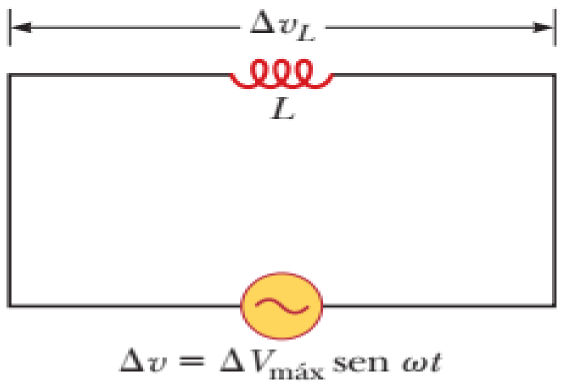
$$\Delta v = L\frac{di_L}{dt} = \Delta V_{max}\sin\omega t$$

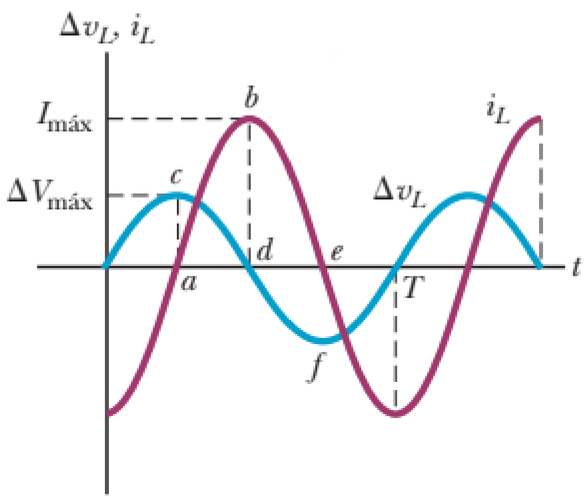
$$di_L = \frac{\Delta V_{max}}{L}\Delta V_{max}\sin\omega t\ \ dt$$

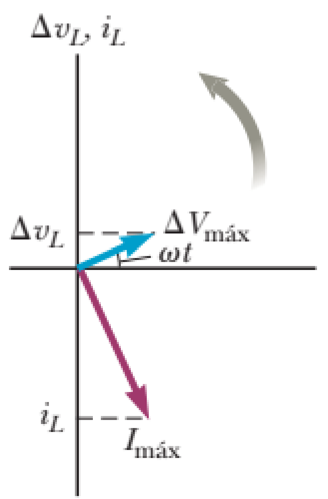
$$i_L = \frac{\Delta V_{max}}{L}\int \sin\omega t\,dt = -\frac{\Delta V_{max}}{\omega L}\cos\omega t$$

$$\cos\omega t = -\sin\left(\omega t - \frac{\pi}{2}\right)$$

$$i_L = \frac{\Delta V_{max}}{\omega L}\sin\left(\omega t - \frac{\pi}{2}\right)$$







Inductores en un circuito de CA

$$I_{max} = \frac{\Delta V_{max}}{\omega L}$$

$$X_L = \omega L = 2\pi f L$$

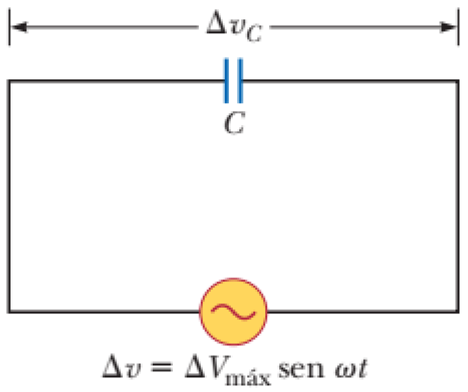
Reactancia inductiva

$$I_{max} = \frac{\Delta V_{max}}{X_L}$$

$$\Delta v_L = -L\frac{di_L}{dt} = \Delta V_{mac} \sin \omega t = -I_{max} X_L \sin \omega t$$

Capacitores en un circuito de CA

$$\Delta v - \Delta v_C = 0 \qquad \Delta v - \frac{q}{C} = 0$$

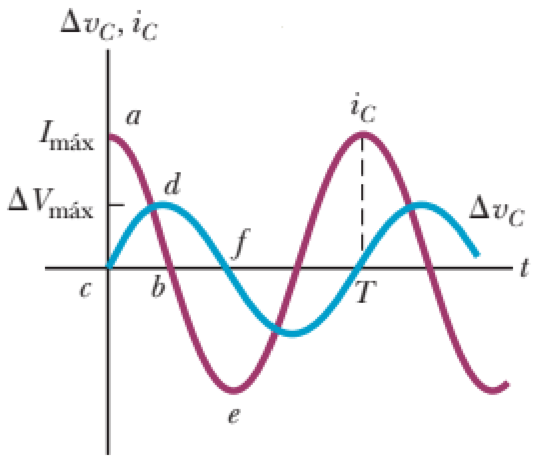


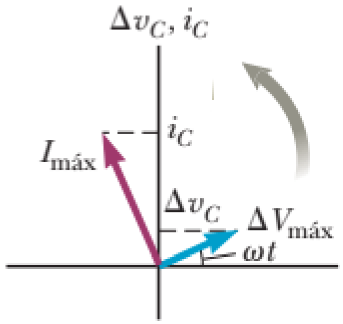
$$q = C\Delta V_{max} \sin \omega t$$

$$i_C = \frac{dq}{dt} = \omega C \Delta V_{max} \cos \omega t$$

$$\cos \omega t = \sin\left(\omega t + \frac{\pi}{2}\right)$$

$$i_C = \omega C \Delta V_{max} \sin\left(\omega t + \frac{\pi}{2}\right)$$





Capacitores en un circuito de CA

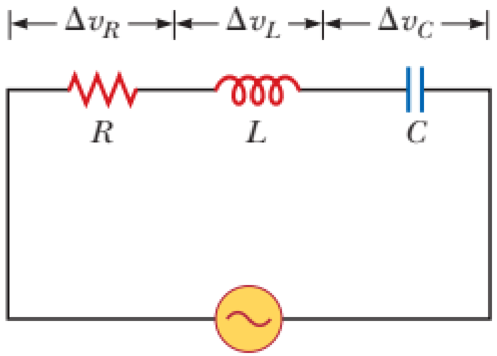
$$I_{max} = \omega C \Delta V_{max} = \frac{\Delta V_{max}}{\frac{1}{\omega C}}$$

$$X_C = \frac{1}{\omega C} = \frac{1}{2\pi f C}$$ Reactancia capacitiva

$$I_{max} = \frac{\Delta V_{max}}{X_C}$$

$$\Delta v_C = \Delta V_{max} \sin \omega t = I_{max}\, X_C \sin \omega t$$

Circuito RLC en serie

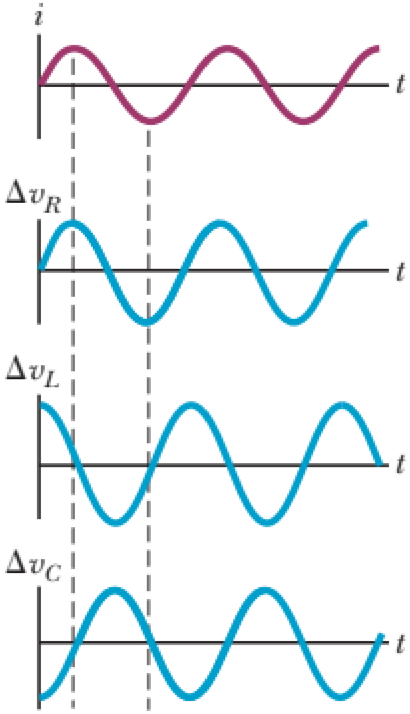


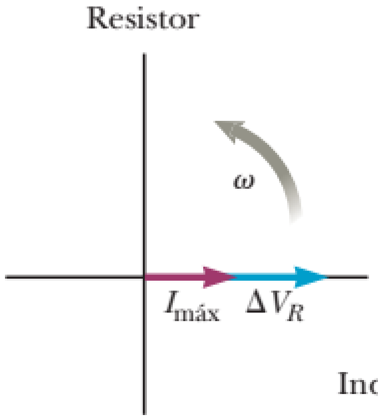
$$i = I_{max}\sin(\omega t + \phi)$$

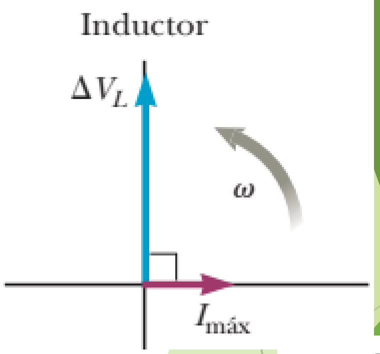
$$\Delta v_R = I_{max} R \sin\omega t = \Delta V_R \sin\omega t$$

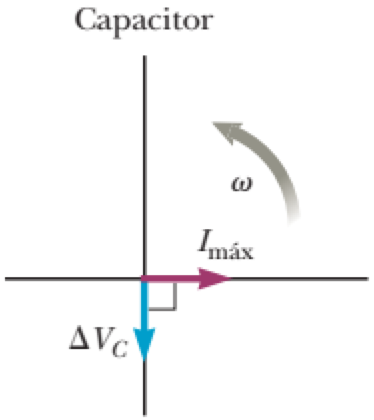
$$\Delta v_L = I_{max} X_L \sin\left(\omega t + \frac{\pi}{2}\right) = \Delta V_L \cos\omega t$$

$$\Delta v_C = I_{max}\, X_C \sin\left(\omega t - \frac{\pi}{2}\right) = -\Delta V_C \cos\omega t$$





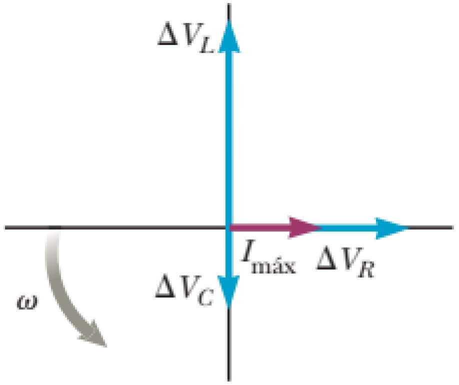


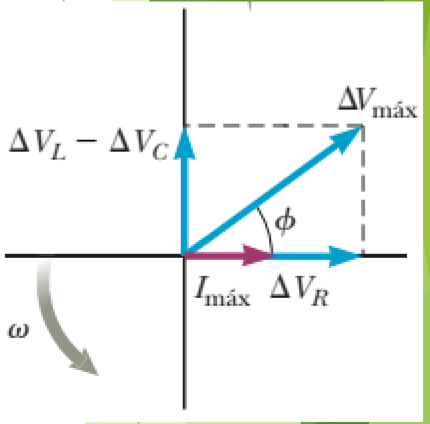


Circuito RLC en serie

$$\Delta V_{max} = \sqrt{\Delta V_R^2 + (\Delta V_L - \Delta V_C)^2} = \sqrt{(I_{max}R)^2 + (I_{max}X_L - I_{max}X_C)^2} = I_{max}\sqrt{R^2 + (X_L - X_C)^2}$$

$$I_{max} = \frac{\Delta V_{max}}{\sqrt{R^2 + (X_L - X_C)^2}}$$





$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$ Impedancia

$$I_{max} = \frac{\Delta V_{max}}{Z}$$

$$\phi = arc\,\text{tg}\left(\frac{\Delta V_L - \Delta V_C}{\Delta V_R}\right) = arc\,\text{tg}\left(\frac{I_{max}X_L - I_{max}X_C}{i_{max}R}\right) \qquad \phi = arc\,\text{tg}\left(\frac{X_L - X_C}{R}\right)$$

Circuito RLC en serie

$$i = \sqrt{2}\, I \sin(\omega t + \phi) \qquad I_{ef} = \frac{I_{máx}}{\sqrt{2}}$$

$$\Delta v = \sqrt{2}\, \Delta V \sin \omega t$$

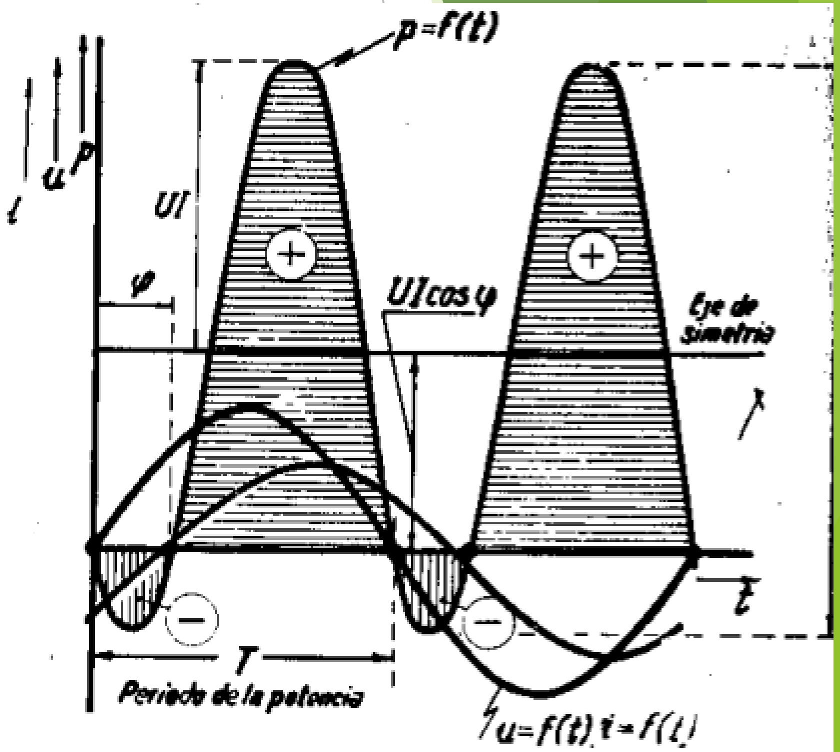
$$p = \Delta V . i = \sqrt{2}\, \Delta V\ I \sin \omega t\ \sqrt{2}\, I \sin(\omega t + \phi) = 2\Delta V\ I \sin(\omega t + \phi) \sin \omega t$$

$$= 2\Delta V\ I\ [sin^2\ \omega t \cos \phi + \sin \omega t \cos \omega t \sin \varphi]$$

$$sin^2 \omega t = \frac{1 - \cos 2\omega t}{2} \qquad \sin \omega t \cos \omega t = \frac{\sin 2\omega t}{2}$$

$$p = \Delta V\ I [\cos \phi - \cos 2\omega t \cos \phi + \sin 2\omega t \sin \phi]$$

$$p = \Delta V\ I \cos \phi - \Delta V\ I \cos\ (2\omega t\ \pm \phi)$$



$$P = \frac{1}{T}\int_0^T p\, dt = \frac{1}{T} \Delta V\ I \int_0^T \cos \phi - \cos\ (2\omega t\ \pm \phi) dt = \Delta V\ I \cos \phi$$

Potencia activa

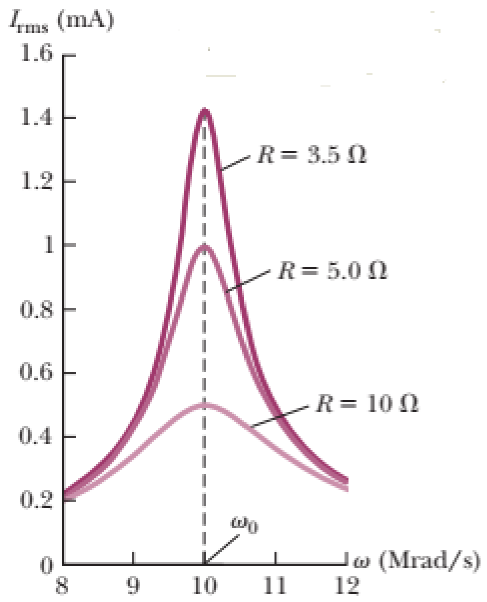
Resonancia en circuito RLC serie

$$I_{máx} = \frac{\Delta V_{ef}}{Z}$$

$$I_{ef} = \frac{\Delta V_{ef}}{\sqrt{R^2 + (X_L - X_C)^2}}$$

$X_L - X_C = 0$ Frecuencia de resonancia

$$\omega_0 = \frac{1}{\sqrt{LC}}$$



Resonancia en circuito RLC serie

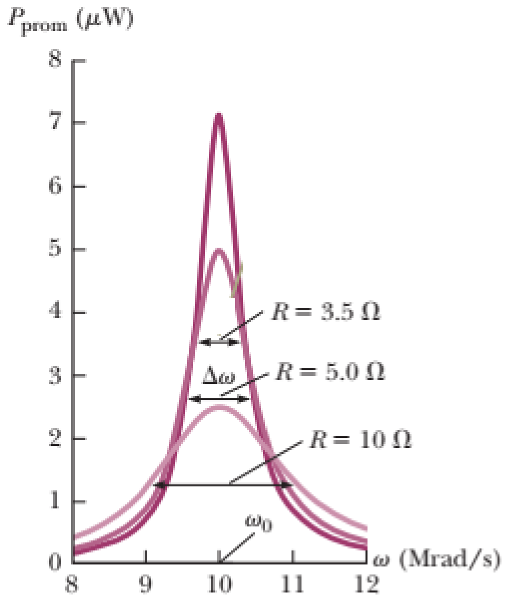
$$P_{promedio} = \frac{(\Delta V_{ef})^2 R\omega^2}{R^2\omega^2 + L^2(\omega^2 - \omega_0^2)^2}$$

$$Q = \frac{\omega_0}{\Delta\omega}$$

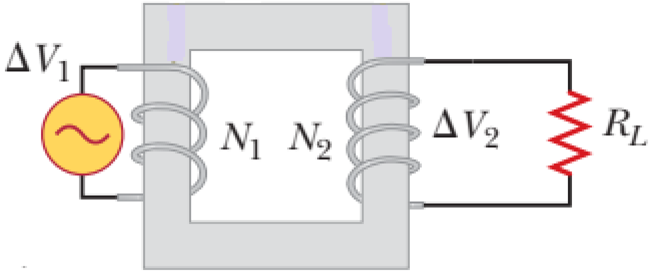
Factor de calidad

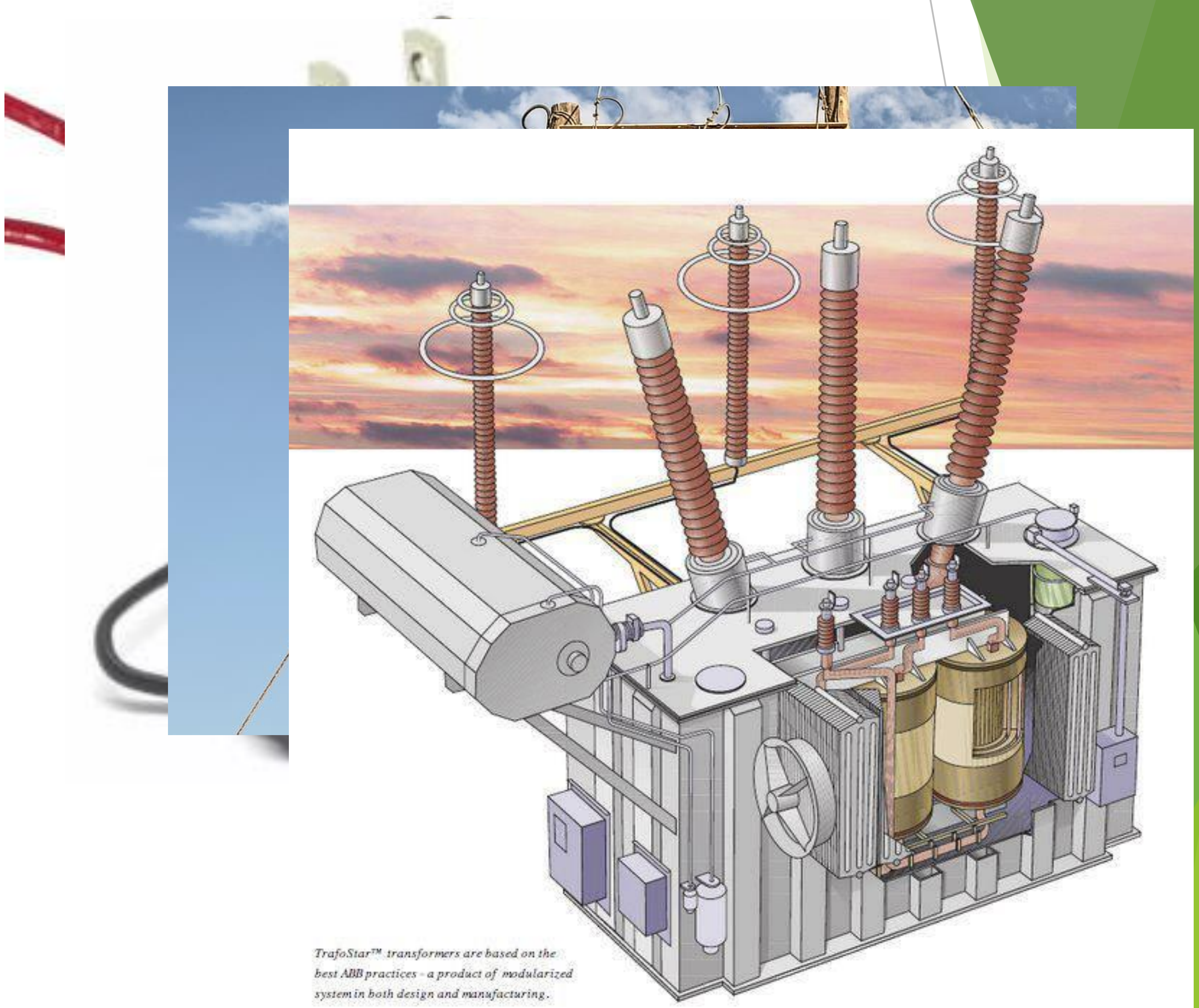
$$\Delta\omega = \frac{R}{L}$$

$$Q = \frac{\omega_0 L}{R}$$



Transformación y transmisión de potencia





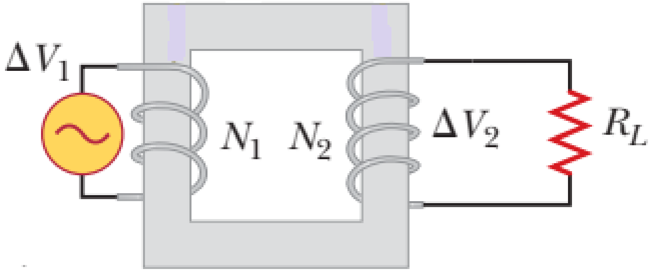
Transformación y transmisión de potencia

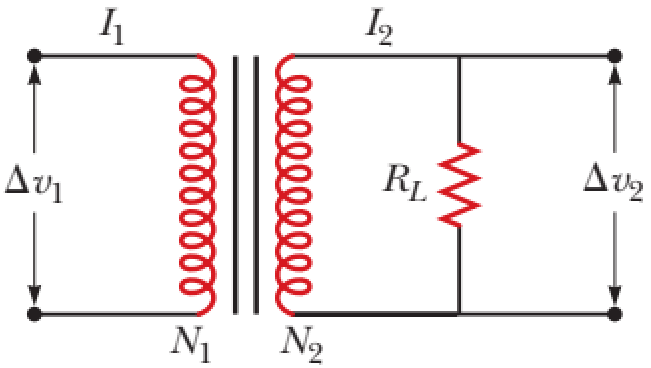
$$\Delta V_1 = -N_1 \frac{d\phi_B}{dt}$$

$$\Delta V_2 = -N_2 \frac{d\phi_B}{dt}$$

$$\frac{\Delta V_1}{\Delta V_2} = \frac{N_1}{N_2} \Rightarrow \Delta V_1 = \frac{N_1}{N_2} \Delta V_2$$

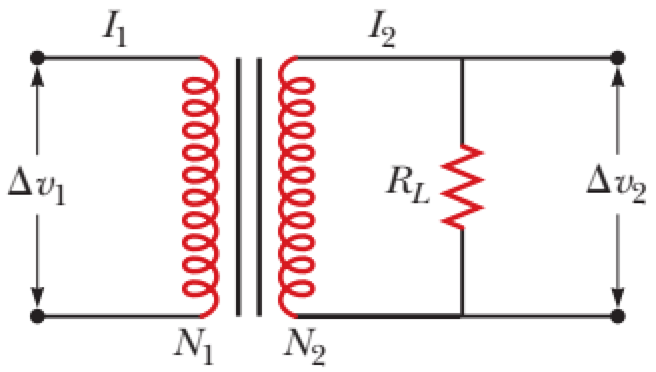
$$I_1 \Delta V_1 = I_2 \, \Delta V_2$$



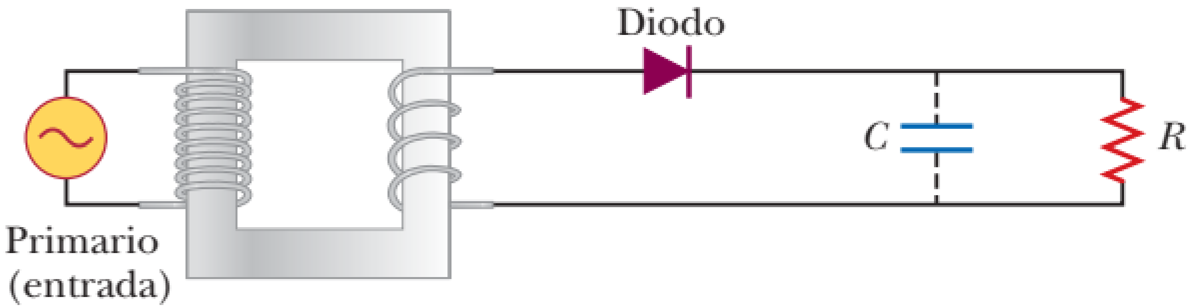


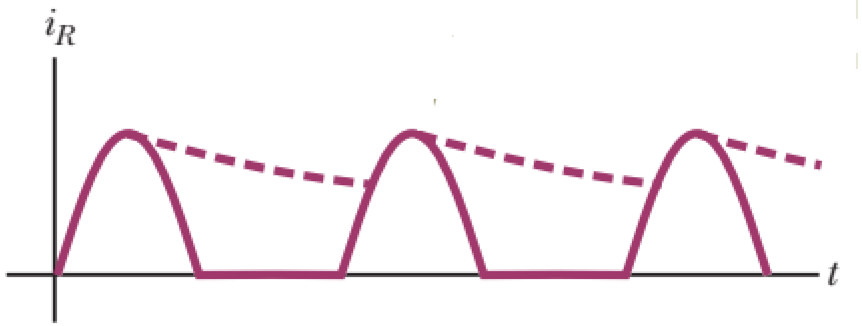
Transformación y transmisión de potencia

$$R_{eq} = \left(\frac{N_1}{N_2}\right)^2 R_L$$



Transformación y transmisión de potencia





Instalaciones eléctricas domiciliarias