Unidad 6: Ondas Electromagnéticas

Ecuaciones de Maxwell

Ecuaciones de Maxwell

$$\oint \vec{E}.d\vec{A} = \frac{q}{\varepsilon_0}$$

Relaciona un campo eléctrico con la distribución de carga que lo produce

$$\oint \vec{B}.\, d\vec{A} = 0$$

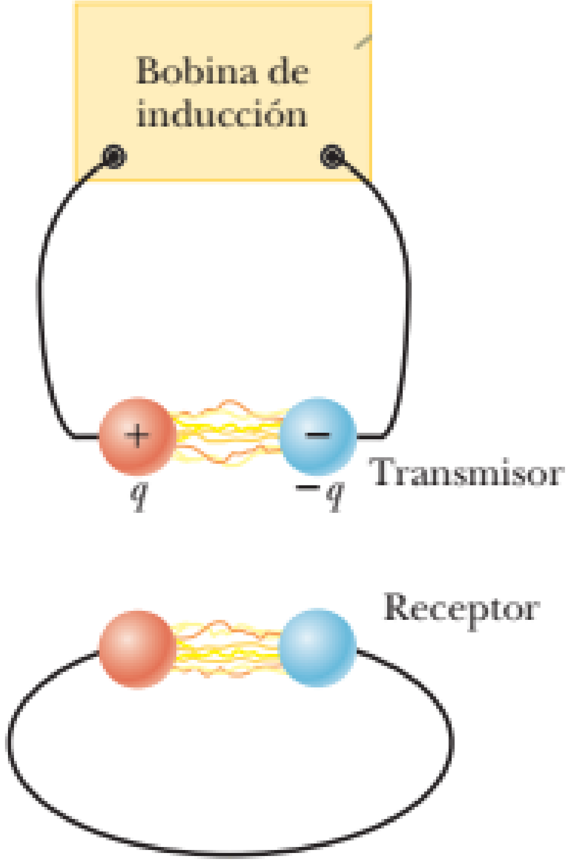
La cantidad de líneas de campo magnético que ingresar a un volumen cerrado son las mismas que egresan

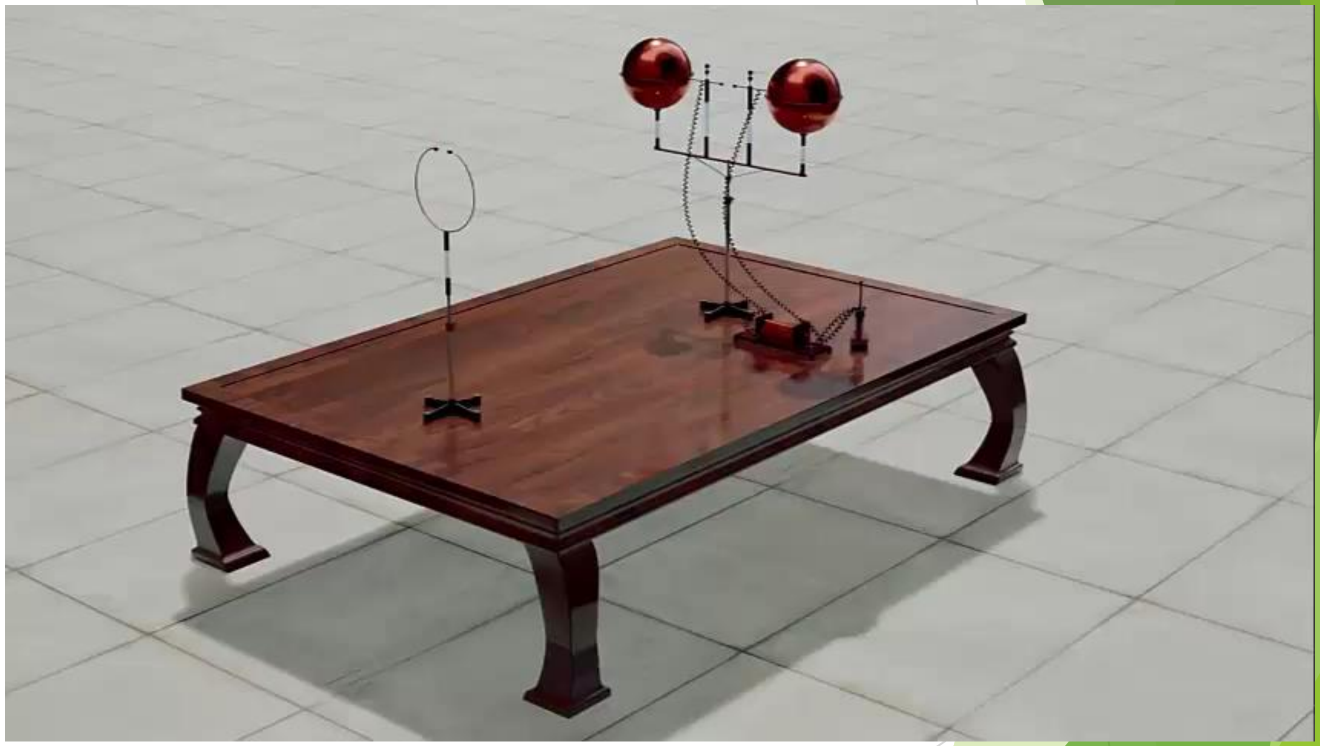
$$\oint \vec{E}.d\vec{s} = -\frac{d\phi_B}{dt}$$

Relaciona un campo eléctrico con un campo magnético variable

$$\oint \vec{B}.d\vec{s} = \mu_0 I + \epsilon_0\mu_0\frac{d\phi_E}{dt}$$

Experimentos de Hertz

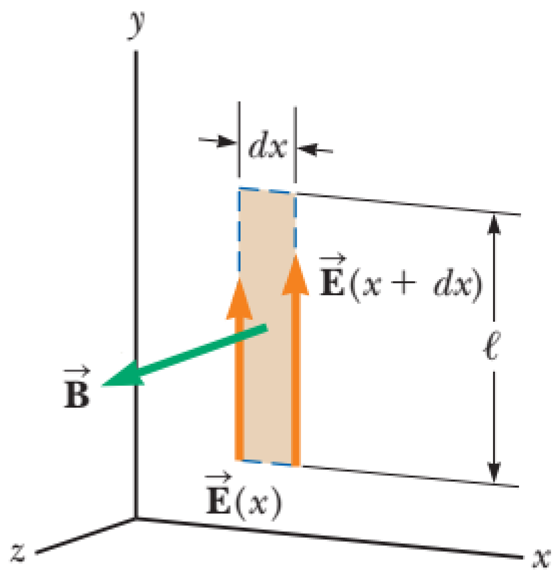


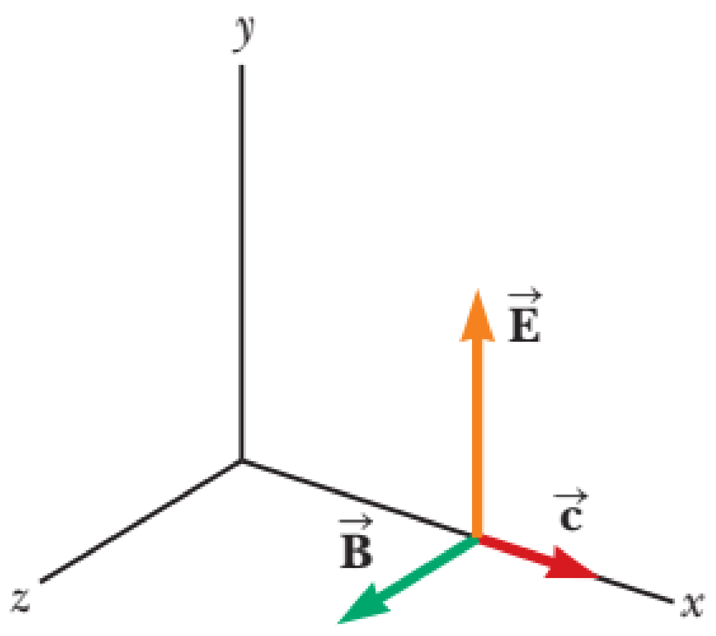


Ondas electromagnéticas planas

$$\oint \vec{E}.d\vec{s} = -\frac{d\phi_B}{dt}$$

$$E(x+dx) \approx E(x) + \left.\frac{dE}{dx}\right|_{l:cte} dx \ = \mathrm{E}(x) + \frac{\partial E}{\partial x}dx$$



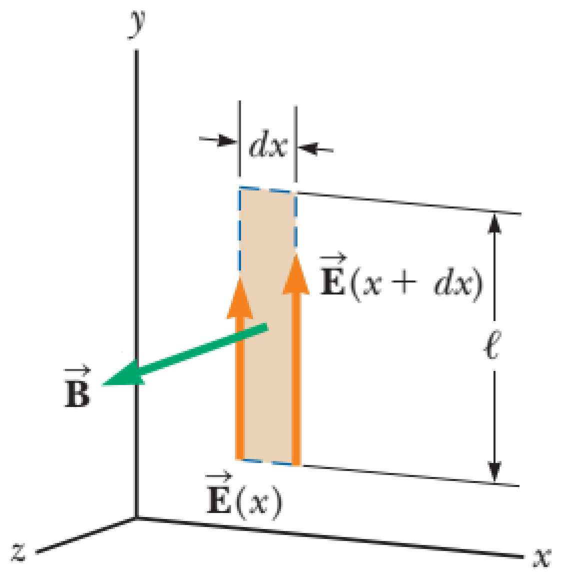


Ondas electromagnéticas planas

$$\oint \vec{E}.d\vec{s} = [E(x+dx)]l - [E(x)]l \approx l\left(\frac{\partial E}{\partial x}\right)dx$$

$$\frac{d\phi_B}{dt} = l\,dx\,\frac{dB}{dt}\bigg|_{x:cte.} = l\,dx\,\frac{\partial B}{\partial t}$$

$$l\left(\frac{\partial E}{\partial x}\right)dx = -l dx\,\frac{\partial B}{\partial t} \quad \Rightarrow \quad \frac{\partial E}{\partial x} = -\frac{\partial B}{\partial t}$$



Ondas electromagnéticas planas

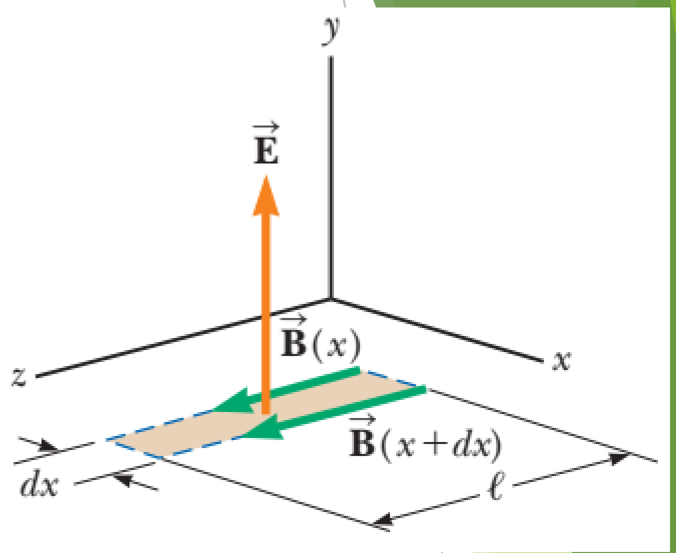
$$\oint \vec{B}.d\vec{s} = [B(x)]l - [B(x+dx)]l \approx -l\left(\frac{\partial B}{\partial x}\right)dx$$

$$\phi_E = Eldx \qquad \frac{d\phi_E}{dt} = l\,dx\,\frac{\partial E}{\partial t}$$

$$-l\left(\frac{\partial B}{\partial x}\right)dx = \mu_0\varepsilon_0 l\,dx\left(\frac{\partial E}{\partial t}\right) \Rightarrow \frac{\partial B}{\partial x} = -\mu_0\varepsilon_0\frac{\partial E}{\partial t}$$

$$\frac{\partial^2 E}{\partial x^2} = \mu_0\varepsilon_0\frac{\partial^2 E}{\partial t^2} \qquad \frac{\partial^2 B}{\partial x^2} = \mu_0\varepsilon_0\frac{\partial^2 B}{\partial t^2}$$

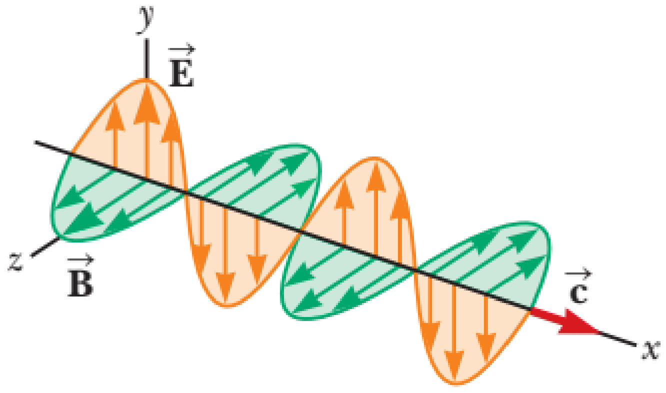
$$c = \frac{1}{\sqrt{\mu_0\varepsilon_0}}$$



Ondas electromagnéticas planas

Si suponemos que los campos eléctrico y magnético se representan con funciones senoidales:

$$\mathrm{E} = E_{máx}\cos(kx - \omega t) \qquad \mathrm{B} = B_{máx}\cos(kx - \omega t)$$



$$k = \frac{2\pi}{\lambda} \qquad \lambda\text{: }longitud\ de\ onda$$

$$\frac{\omega}{k} = \frac{2\pi f}{\frac{2\pi}{\lambda}} = \lambda f = c$$

$$\frac{\partial E}{\partial x} = -kE_{máx}\sin(kx - \omega t)$$

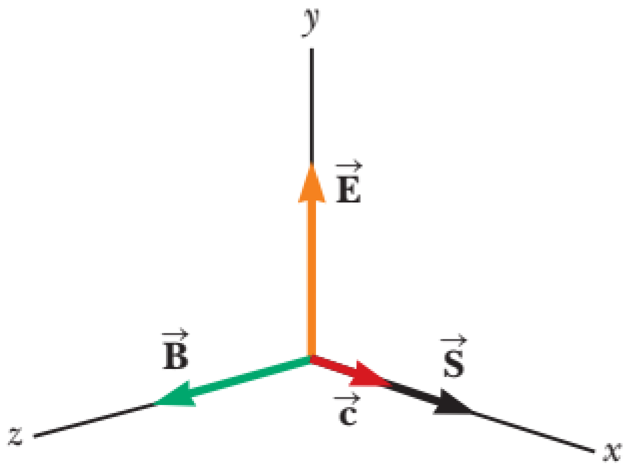
$$\frac{\partial B}{\partial t} = \omega B_{máx}\sin(kx - \omega t)$$

$$\Rightarrow \frac{\partial E}{\partial x} = -\frac{\partial B}{\partial t} \Rightarrow kE_{máx} = \omega B_{máx} \Rightarrow \frac{E_{máx}}{B_{máx}} = \frac{\omega}{k} = \mathrm{c}$$

Energía transportada por las ondas electromagnéticas

$$\vec{S} \equiv \frac{1}{\mu_0}\vec{E} \times \vec{B}$$

$\vec{S}$: vector de Poynting



$$|\vec{E} \times \vec{B}| = E.B \quad \Rightarrow \quad S = \frac{E.B}{\mu_0}$$

$$I = S_{promedio} = \frac{E_{máx}.B_{máx}}{2.\mu_0} = \frac{E_{máx}^2}{2\mu_0 c} = \frac{cB_{máx}^2}{2\mu_0}$$

$$u_E = \frac{1}{2}\varepsilon_0 E^2 \qquad B = \frac{E}{c} \qquad c = \frac{1}{\sqrt{\mu_0\varepsilon_0}}$$

$$u_B = \frac{1}{2}\frac{B^2}{\mu_0} = \frac{\left(\frac{E}{c}\right)^2}{2\mu_0} = \frac{\mu_0\varepsilon_0}{2\mu_0}E^2 = \frac{1}{2}\varepsilon_0 E^2 \quad \Rightarrow \quad u_B = u_E = \varepsilon_0 E^2 = \frac{B^2}{\mu_0}$$

$$u_{promedio} = \varepsilon_0 E_{máx}^2 = \frac{1}{2}\varepsilon_0 E_{máx}^2 = \frac{B_{máx}^2}{2\mu_0} \quad \Rightarrow \quad I = S_{promedio} = cu_{promedio}$$

Producción de ondas electromagnéticas por una antena